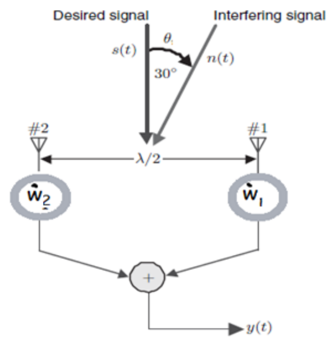
Benha University	**Antennas &Wave Propagation**	**Electrical Eng. Dept.**
Faculty of Engineering	2^{nd}_part	**4^{th} year communication**
Shoubra	-SHEET (4)	**2020-2021**

1. The field pattern of an antenna array is giving by
   $f(\theta, \varphi)=1+\cos\theta$
   (a) Draw radiation pattern of this array
   (b) Find the directivity.

2. For two elements array with spacing d and progressive phase shift β find:
   a. d and β to have max $\theta=0^o$ *and null at* $\theta=60^o$
   b. Condition of spacing d for having no nulls while maintaining max at $\theta=0^o$

3. Design a uniform array with minimum number of elements and no grating lobes such that the array max radiation at $\theta=0^o$ with *AF* of side lobes <0.26 then draw the radiation pattern.

4. Determine complex weights of a two elements linear array, half wavelength apart to receive a desired signal s(t) at $\theta=0^o$ while tunning out an interferer *n(t) at* $\theta=30^o$ as shown in figure. The elements of the array are assumed to be isotropic for simplicity



5. Prove that a planar $M \times N$ array has an array factor equal to

$$AF_n(\theta,\phi) = \left\{ \frac{1}{M} \frac{\sin\left(\frac{M}{2}\psi_x\right)}{\sin\left(\frac{\psi_x}{2}\right)} \right\} \left\{ \frac{1}{N} \frac{\sin\left(\frac{N}{2}\psi_y\right)}{\sin\left(\frac{\psi_y}{2}\right)} \right\}$$

where

$$\psi_x = kd_x \sin\theta\cos\phi + \beta_x$$
$$\psi_y = kd_y \sin\theta\sin\phi + \beta_y$$

GOOD LUCK *Dr. Gehan Sami*

1/ Sheet # Antenna part 2 sheet 4

(1) The field pattern of an array is given by

$$F(\theta,\phi) = 1+\cos\theta$$

1- Draw Radiation pattern of this Array

2- Find Directivity of this Array

Soln

To Draw Pattern Find Nulls & Max

Nulls $1+\cos\theta_{null}=0$ @ $\theta_{null}=\cos^{-1}(-1)=\pi$

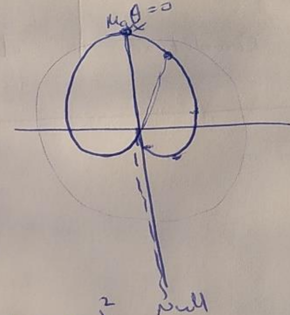
Max $|1+\cos(\theta_{max})|=2$ or $\theta_{max}=\cos^{-1}1=0$

Max of cos = 1

$0<\theta<\pi$

Use Table fn

θ	$1+\cos\theta$
$\theta=0$	2
0.2π	1.8
0.4π	1.3
0.6π	0.7
0.8π	0.2
π	0



Directivity $P_n = \left[\frac{1+\cos\theta}{2}\right]^2 = \frac{1}{4}(1+\cos\theta)^2$

$$D = \frac{4\pi}{2\pi\int_0^{\pi}\frac{1}{4}(1+\cos\theta)^2\sin\theta\,d\theta} = \frac{8}{2.67} = 3$$

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(2) for 2 Element array with spacing d and progressive phase shift β

Find (a) d and β to have θ_{max} @ $=0^\circ$ and null at $\theta=60^\circ$

(b) condition of spacing d for having no nulls while maintaining max @ 0°

Soln $AF = \frac{\sin 2\psi/2}{2\sin\psi/2}$ $\quad \psi = kd\cos\theta+\beta$

Max @ $\psi=0$: $\beta = -kd\cos\theta_{max}$ $\quad \beta = -\frac{2\pi}{\lambda}d$

Null at $2\frac{\psi}{2}=\pm\pi$ or $kd(\underbrace{\cos\theta_{null}}_{0.5}-1)=\pm\pi \rightarrow \frac{2\pi}{\lambda}d = 2\pi \rightarrow \boxed{d=\lambda}$

$\boxed{\beta=-2\pi}$

(b) Find condition for d to have No nulls and keep max at $\theta = 0°$

Soln Max at 0° with new d_1 $\beta = -kd_1 \cos 0 = -kd_1$

Null at $\frac{N\psi}{2} = \pm m\pi$ ($m = 1, 2, 4, \dots$) $N = 2$

$$\psi = \pm\pi$$

$$kd_1 \cos\theta_{null} - kd_1 = \pm\pi$$

$$(\cos\theta_{null} - 1) = \pm\frac{\pi\lambda}{2\pi d_1}$$

or $$\theta_{null} = \cos^{-1}\left(1 \pm \frac{\lambda}{2d_1}\right)$$

for No Nulls $\rightarrow$ $\left|1 \pm \frac{\lambda}{2d_1}\right| > 1$

$-1 \mp \frac{\lambda}{2d_1} > 1 \rightarrow \mp\frac{\lambda}{2d_1} > 2$ or $d_1 < \lambda/4$

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(3) Design a Uniform array with min no. of Elements & No grating lobes such that the array max radiation $\theta = 0$ (End fire) with $AF|_{sidelobe} < 0.26$ then Draw Radiation Pattern

Soln Side lobe occure at $N\frac{\psi}{2} = \frac{3\pi}{2}$

$AF = \dfrac{\sin\frac{N\psi}{2}}{N\sin\frac{\psi}{2}}$ $\therefore AF = \dfrac{\sin\left(\frac{3\pi}{2}\right)}{N\sin\left(\frac{3\pi}{2N}\right)} < 0.26$

$N\sin\left(\frac{3\pi}{2N}\right) > \left|\frac{-1}{0.26}\right|$ $N\sin\left(\frac{3\pi}{2N}\right) > |-3.84|$

for $N = 2$ $2\sin\left|\frac{3\pi}{4}\right| = 1.4$ refused

$N = 3$ $3\sin\left(\frac{3\pi}{6}\right) = 3$ refused

$N = 4$ $4\sin\left(\frac{3\pi}{8}\right) = 3.69$ refused

$N = 5$ $5\sin\left(\frac{3\pi}{10}\right) = 4.04$ ✓

$\therefore$ Min number of Element $N = 5$

It is required to have No grating lobe $\frac{\psi}{2} = \pm\pi$ $\beta = -kd\cos 0 = -kd$

$\frac{2\pi}{\lambda} d(\cos\theta_g - 1) = \pm 2\pi$ $\theta_g = \cos^{-1}\left(1 \pm \frac{\lambda}{d}\right)$ $\left|1 \pm \frac{\lambda}{d}\right| > 1$

$\lambda/d > 2 \rightarrow d < \lambda/2 \xrightarrow{\text{Set}} d = 0.4\lambda$ for ex.

$\therefore \beta = -\frac{2\pi}{\lambda} * 0.4\lambda = -0.8\pi$

Plot AF Pattern

$N = 5 \quad d = 0.4\lambda \quad \beta = -0.8\pi \qquad kd = \frac{2\pi}{\lambda} \times 0.4\lambda = 0.8\pi$

Soln

Find Nulls & Max $\longrightarrow$ max @ $\theta = 0^\circ$

Nulls @ $\frac{N\psi}{2} = \pm \underbrace{m}_{\neq 0,5,\ldots}\pi$

$\cancel{kd}\ 0.8\pi\,(\cos\theta_{null} - 1) = \pm \frac{2m\pi}{5}$

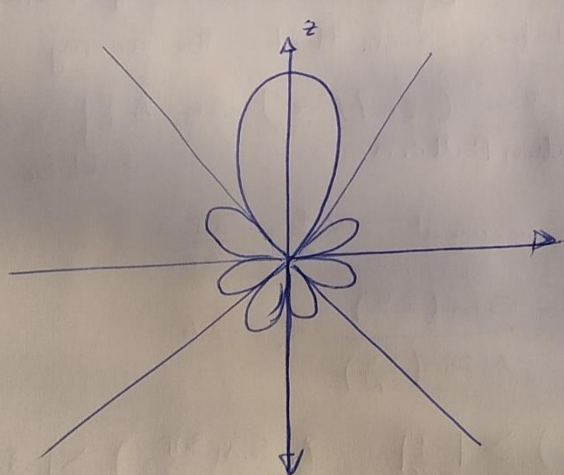
$\theta_{null} = \cos^{-1}\left(1 \pm \frac{m}{2}\right)$
- $m=1$: $\cos^{-1} \frac{1}{2} \rightarrow 60^\circ$
- $m=2$: $\cos^{-1} 0 \rightarrow 90^\circ$
- $m=3$: $\cos^{-1} -\frac{1}{2} \rightarrow 120^\circ$
- $m=4$: $\cos^{-1} -1 \rightarrow 180^\circ$

Max للتأكيد @ $\frac{\psi}{2} = \pm \underbrace{m}_{0,1,2,3}\pi$ @ $m=0$ $\theta_{max} = 0^\circ$

$0.8\pi\,(\cos\theta_{max} - 1) = \pm 2\pi$

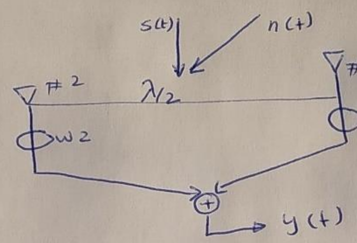
$\theta_{max} = \cos^{-1}(1 \pm 2.5) \longrightarrow \times$

So no grating lobe only one Max exist



3/

(4) Determine complex weights of a two Element linear array, half wavelength apart to receive a desired signal of certain magnitude Unity at $\theta_0 = 0^\circ$ while tunning out an interferer (SNOI) at $\theta_1 = 30^\circ$, as shown in Figure. The elements of the array are assumed to be for simplicity isotropic.



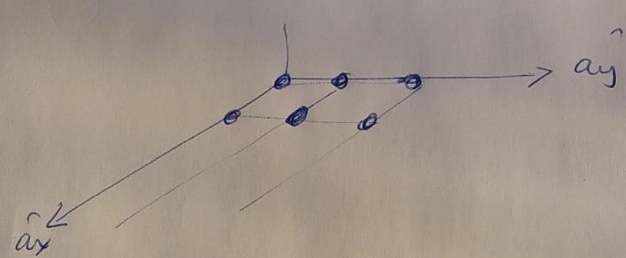
<u>Soln</u> as in lect.

(5) Prove that for planar array $M \times N$ Elements the total array factor equal to $AF = (AF)_x * (AF)_y$

$$= \frac{\sin \frac{M\psi_x}{2}}{M \sin \frac{\psi_x}{2}} * \frac{\sin \frac{N\psi_y}{2}}{N \sin \frac{\psi_y}{2}}$$

Where

$$\psi_x = kd_x \sin\theta \cos\phi + \beta_x$$
$$\psi_y = kd_y \sin\theta \sin\phi + \beta_y$$



take 2X3 planar array as Example.

<u>Soln.</u> as in lect.